Received: *October 18, 2000*, Accepted: *December 18, 2000*

Current correlators in the Coulomb branch of $\mathcal{N} = 4$ SYM

Andreas Brandhuber[*]
Theory Division, CERN, CH-1211 Geneva 23, Switzerland
E-mail: `brandhu@mail.cern.ch`

Konstadinos Sfetsos
Institut de Physique, Université de Neuchâtel, Breguet 1
CH-2000 Neuchâtel, Switzerland
E-mail: `sfetsos@mail.cern.ch`

JHEP12(2000)014

Abstract: We study correlators of $\mathcal{R}$-symmetry currents in the Coulomb branch of $\mathcal{N} = 4$ supersymmetric gauge theory in the large-N limit, using the AdS/CFT correspondence. In particular, we consider gauge fields in the presence of gravity and scalar fields parameterizing the coset $\mathrm{SL}(6, \mathbb{R})/\mathrm{SO}(6)$ in the context of five-dimensional gauged supergravity. From a ten-dimensional point of view these backgrounds correspond to continuous D3-brane distributions. We find the surprising result that all 2-point functions of gauge currents fall into the same universality class, irrespectively of whether they correspond to broken or unbroken symmetries. We show that the problem of finding the spectrum can be mapped into an equivalent Schrödinger problem for supersymmetric quantum mechanics. The corresponding potential is the supersymmetric partner of the potential arising in studies of the spectrum for massless scalars and transverse graviton fluctuations in these backgrounds and the associated spectra are also identical. We discuss in detail two examples where these computations can be done explicitly as in the conformal case.

Keywords: AdS/CFT Correspondance, Supergravity Models.

[*]Since 30th September 2000: Department of Physics, CalTech, Pasadena, CA 91125

Contents

1. Introduction

For several years the dynamics of branes in string theory have been a fruitful playground to test strong coupling physics of gauge theories. For instance, the AdS/CFT correspondence [1, 2, 3] provides us with precise prescriptions to calculate correlation functions, spectra of gauge invariant operators, Wilson loops and c-functions in $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) theory in four dimensions at large-N and large 't Hooft coupling. The data obtained this way from supergravity can sometimes be compared with field theory or provide non-trivial predictions for strongly coupled field theories. This correspondence can be extended also to theories with spontaneously or manifestly broken superconformal symmetry. Such theories arise either by giving vacuum expectation values to fields [1], [5]–[13] or by deforming the conformal theory with relevant operators [14]–[29]. Many of these deformations can be treated efficiently in the context of five-dimensional gauged supergravity [30, 31] and the resulting backgrounds have four-dimensional Poincaré invariance and approach AdS_5 in the ultraviolet (in a field theory terminology). Typically, towards the infrared, singularities appear which are not fully understood and seem to require a proper inclusion of the string theory dynamics or the use of other methods developed in gravity.

JHEP12(2000)014

In this letter we study correlation functions of $\mathcal{R}$-symmetry currents using the holographic description of large-N gauge theories. For the conformal case correlation functions for operators in various representations of the $\mathcal{R}$-symmetry group $\mathrm{SU}(4) \simeq \mathrm{SO}(6)$ have been worked out in great detail (see, for instance, [32, 33]). Less is known about correlators in deformed gauge theories which are described by more general domain wall solutions of gauged supergravity. So far mainly scalars have been studied, namely the minimally coupled scalar [9, 10, 34] (which has the same equation as the transverse traceless graviton modes [35]), active and inert scalars which parameterize deformations of the S^5 [36, 37, 38], but also fermionic and abelian vector field fluctuations for the $\mathcal{N} = 1$ flow of [20] and the $\mathcal{N} = 4$ Coulomb branch background of [9, 10] have been considered recently in [38].

We will show that for a specific class of examples this analysis can be extended to include fluctuations of non-abelian gauge fields which are dual to $\mathcal{R}$-symmetry currents of the gauge theory. We make a general connection between the fluctuation equation and supersymmetric quantum mechanics and find that, the relevant Schrödinger potential, associated with the spectrum, is just the supersymmetric partner of the potential arising from the corresponding massless scalar and transverse graviton-fluctuations equations. We show also that the corresponding spectra are identical. It seems plausible to us that this can be extended to the full set of fields in the supergravity multiplet. Using the AdS/CFT correspondence we calculate two-point functions of the symmetry currents in $\mathcal{N} = 4$ SYM on the Coulomb branch in two particular cases.[1] As expected, we find deviations from the conformal $1/r^6$ fall-off for large separations r. From the non-analytic part of the correlator in momentum space we get contributions that are suppressed exponentially for large separation.

The choice of a particular state on the Coulomb branch breaks the $\mathcal{R}$-symmetry to a subgroup and therefore one might expect that broken and unbroken currents behave differently and in particular one would expect Goldstone bosons corresponding to the broken symmetry. From the dual supergravity point of view this symmetry is a local gauge symmetry and the massless bosons simply get eaten by the gauge fields and make them massive via the Higgs mechanism. Although the equations for broken and unbroken currents look quite different — they correspond, respectively, to massless gauge fields in a curved background and massive gauge fields — the associated spectra are identical. This result is not too surprising since on the Coulomb branch only conformal symmetry is broken but the currents still reside in the same supersymmetry multiplet. However, a small puzzle remains since the correlator has also an analytic piece that depends on which of the broken or unbroken currents are considered. For the two-point function of scalars such analytic terms give rise to contact terms and are usually dropped, but in the case of gauge field correlators they

[1] Other studies of the Coulomb branch of the $\mathcal{N} = 4$ SYM theory using the AdS/CFT correspondence can be found in [39].

JHEP12(2000)014

give rise to terms of the form $x_\mu x_\nu/r^6$, which might be interpreted in field theory as arising from Goldstone bosons. However, we do not find a one to one relation between broken currents and the presence of these terms in the correlators. We believe that these analytic terms are unphysical, since the corresponding mode is non-normalizable, and should be dropped.

The organization of this paper is as follows: in section 2 we present some background material on gauged supergravity and calculation of correlators in AdS/CFT. We also make a general connection between the fluctuation equation and supersymmetric quantum mechanics. In section 3 we focus on our two main examples where calculations can be performed explicitly. We obtain the exact fluctuation spectrum of gauge fields, and the two-point functions in momentum and position space. In section 4 we give a summary of our results and give some final remarks.

2. Generalities

Our starting point is a specific truncation of the $\mathcal{N}=8$ gauged supergravity action [30, 31] including SO(6) gauge fields $A^{ij}_{\widehat{\mu}}$, antisymmetric in i,j, with field strength $F^{ij}_{\widehat{\mu}\widehat{\nu}}$, where $\widehat{\mu},\widehat{\nu}=1,2,3,4,z$; unhatted indices $\mu,\nu=1,2,3,4$ will be used later to denote euclidean directions along the boundary at $z=0$. For notational convenience we will occasionally use the collective index $a=1,2,\ldots,15$ to denote the adjoint representation of SO(6), instead of i and j or we will omit such an index all together. Furthermore, scalars in the $\mathbf{20'}$ are represented by a symmetric traceless matrix M^{ij}. The action of the supergravity truncated to these fields has been constructed in [40] and we follow closely their conventions.

The lagrangian density for the relevant fields of five-dimensional gauged supergravity is

$$\mathcal{L}=\mathcal{L}_{\text{scalar}}+\mathcal{L}_{\text{gauge}}\,, \tag{2.1}$$

where $\mathcal{L}_{\text{scalar}}$ refers to the pure gravity-scalar sector and $\mathcal{L}_{\text{gauge}}$ contains the gauge fields and their interaction with the scalars and gravity. We first recall some results for the pure gravity-scalar sector since we are interested to study fluctuations of the gauge fields in the background of specific solutions of the gravity-scalar sector. The explicit form of the lagrangian is

$$\frac{1}{\sqrt{g}}\mathcal{L}_{\text{scalar}}=\frac{1}{4}\mathcal{R}-\frac{1}{16}\operatorname{Tr}\left(\partial_{\widehat{\mu}}MM^{-1}\partial^{\widehat{\mu}}MM^{-1}\right)-P\,, \tag{2.2}$$

where the potential is

$$P=-\frac{g^2}{32}\left[(\operatorname{Tr}M)^2-2\operatorname{Tr}(M^2)\right], \tag{2.3}$$

with g being a mass scale. Alternatively we may use the length scale R via the relation $g=2/R$.

Supersymmetric solutions of (2.2) preserving 16 supercharges and Poincaré symmetry in four-dimensions have been studied extensively and they correspond to states on the Coulomb branch of $\mathcal{N}=4$ SYM theory. Their interpretation in ten dimensions is simply in terms of a continuous distribution of D3-branes. For these backgrounds the matrix of scalar fields can be brought to a diagonal form using a gauge transformation. Thus we are left with six scalar fields that parameterize

$$M = \mathrm{diag}(e^{2\beta_1}, \ldots, e^{2\beta_6})\,, \tag{2.4}$$

obeying the constraint $\sum_{i=1}^{6} \beta_i = 0$. There are five independent scalar fields, denoted by α_I, $I = 1, 2, \ldots, 5$, and the relation to the β_i's is given by $\beta_i = \sum_{I=1}^{5} \lambda_{iI}\alpha_I$, where λ_{iI} is a 6×5 matrix, with rows corresponding to the fundamental representation of $\mathrm{SL}(6, \mathbb{R})$; the normalization conventions can be found in [13, eq. (2.4)]. The metric ansatz reads

$$ds^2 = e^{2A(z)}(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu) = dr^2 + e^{2A(r)}\eta_{\mu\nu}dx^\mu dx^\nu\,, \tag{2.5}$$

where the relation between the coordinates z and r is such that $dr = -e^A dz$. In addition, all scalar fields depend on the variable r or equivalently z. The most general solution preserving 16 supercharges has been found in [11] and is conveniently presented in terms of an auxiliary function $F(g^2 z)$. Specifically, the conformal factor is given by

$$e^{2A} = g^2(-F')^{2/3}\,, \tag{2.6}$$

where the prime denotes the derivative with respect to the argument of $F(g^2 z)$. In addition, the profiles of the scalar fields are

$$e^{2\beta_i} = \frac{f^{1/6}}{F - b_i}\,, \qquad f = \prod_{i=1}^{6}(F - b_i)\,, \qquad i = 1, 2, \ldots, 6\,. \tag{2.7}$$

The constants of integration are ordered as $b_1 \geq b_2 \geq \cdots \geq b_6$ and the function F is constrained to obey the differential equation

$$(F')^4 = f\,. \tag{2.8}$$

Equating n of the integration constants b_i (or equivalently the associated scalar fields β_i) corresponds to preserving an $\mathrm{SO}(n)$ subgroup of the original $\mathrm{SO}(6)$ $\mathcal{R}$-symmetry group. We note in passing, that there is a deep connection between solutions of the gravity-scalar sector of the five-dimensional gauged supergravity that we just reviewed, and the theory of algebraic curves and associated Riemann surfaces to which the differential equation (2.8) is related [11, 13].

Let us now turn to the part of the lagrangian containing the gauge fields. First, we have to replace the partial derivatives in (2.2) by gauge-covariant ones $\partial_{\widehat{\mu}}M^{ij} \to \partial_{\widehat{\mu}}M^{ij} + g(A^{ik}_{\widehat{\mu}}M^{kj} + A^{jk}_{\widehat{\mu}}M^{ik})$, and, second, we add the gauge kinetic term

$$\frac{1}{\sqrt{g}}\mathcal{L}_{\mathrm{gauge}} = -\frac{1}{8}(M^{-1})^{ij}(M^{-1})^{kl}F^{ik}_{\widehat{\mu}\widehat{\nu}}F^{jl\widehat{\mu}\widehat{\nu}}\,. \tag{2.9}$$

Since we are interested in two-point functions we only need to keep terms in (2.1) and (2.9) which are quadratic in the gauge fields and the scalar fluctuations in the symmetric unimodular matrix M. Note that although for our solution the matrix M is diagonal as in (2.4), we have to consider fluctuations along the diagonal as well as off-diagonal ones. Using the fact that M is diagonal (2.4) for our backgrounds, we collect all terms that can give quadratic terms in the fluctuations of the scalars and the gauge fields

$$\begin{aligned}\frac{1}{\sqrt{g}}\mathcal{L}_{\text{quad.}} = & -\frac{1}{8}e^{-2(\beta_i+\beta_j)}F^{ij}_{\widehat{\mu}\widehat{\nu}}F^{\widehat{\mu}\widehat{\nu}}_{ij} - \frac{g^2}{4}\sinh^2(\beta_i-\beta_j)A^{ij}_{\widehat{\mu}}A^{\widehat{\mu}}_{ij} - \\ & -\frac{g}{8}\operatorname{Tr}\Big((\partial_{\widehat{\mu}}MM^{-1}-M^{-1}\partial_{\widehat{\mu}}M)A^{\widehat{\mu}}\Big)\Big|_{\text{quad.}} - \\ & -\frac{1}{16}\operatorname{Tr}(\partial_{\widehat{\mu}}MM^{-1}\partial^{\widehat{\mu}}MM^{-1}) - P|_{\text{quad.}}\,. \end{aligned}\tag{2.10}$$

The first line above is already quadratic in the gauge field fluctuations. We emphasize that $F^{ij}_{\widehat{\mu}\widehat{\nu}} = \partial_{\widehat{\mu}}A^{ij}_{\widehat{\nu}} - \partial_{\widehat{\nu}}A^{ij}_{\widehat{\mu}}$ is, for our purposes, the relevant part of the gauge field strength. The second line in the above expression is already linear in the gauge field fluctuaction. Hence, we are supposed to expand it to linear order in the scalar fluctuations. Finally, the third line has to be expanded to quadratic order in the scalar field fluctuations. In this paper we are only interested in the gauge field fluctuations which, however, couple to fluctuations of the scalars. Therefore, it is not a priori correct to simply keep the terms in the first line in (2.10) and drop the rest. Nevertheless, we will now explain that this procedure gives the correct result since there is a field redefinition that effectively decouples the gauge field fluctuations from those of the scalars.[2] To see that let us expand the second line in (2.10) and keep the linear term in the scalar field fluctuations. We find that

$$\begin{aligned}-\frac{g}{8}\operatorname{Tr}\Big((\partial_{\widehat{\mu}}MM^{-1}-M^{-1}\partial_{\widehat{\mu}}M)A^{\widehat{\mu}}\Big)\Big|_{\text{quad.}} = & \frac{g}{8}\Big((e^{-2\beta_j}-e^{-2\beta_i})\partial_{\widehat{\mu}}\delta M_{ij} + \\ & + 2(e^{-2\beta_i}\partial_{\widehat{\mu}}\beta_j - e^{-2\beta_j}\partial_{\widehat{\mu}}\beta_i)\delta M_{ij}\Big)A^{\widehat{\mu}}_{ij}\,.\end{aligned}\tag{2.11}$$

From this we immediately deduce that the diagonal fluctuations δM_{ii} do not couple to the gauge fields. A less trivial fact is that the scalar fluctuations in δM_{ij} that belong to any unbroken subgroup of SO(6) do not couple to the gauge fields as well. The reason is that in this case $\beta_i = \beta_j$, since then the corresponding integration constants in (2.7) are equal, i.e. $b_i = b_j$. Hence, let us consider the remaining cases with $\beta_i \neq \beta_j$ which arise when the indices i, j belong to the coset. If we make the field redefinition

$$A^{ij}_{\widehat{\mu}} \longrightarrow A^{ij}_{\widehat{\mu}} + \frac{1}{g}\,\partial_{\widehat{\mu}}\left(\frac{\delta M_{ij}}{e^{2\beta_i}-e^{2\beta_j}}\right), \qquad \beta_i \neq \beta_j\,, \tag{2.12}$$

[2]We thank M. Bianchi for prompting us to explain in detail how the decoupling between scalar and gauge field fluctuations actually works as well as for other related comments.

the mixed terms between scalar and gauge field fluctuations in (2.10) (with the substitution (2.11) understood) disappear and the fluctuations decouple. Note that the field redefinition (2.12) acts as an abelian gauge transformation and as such it leaves the gauge field strength $F^{ij}_{\widehat{\mu}\widehat{\nu}}$ invariant (to the quadratic order we are working). We emphasize that the field redefinition (2.12) does not guarantee that there will be no mixing between scalar and gauge field fluctuations at the cubic or at some higher order in the fluctuating fields, but only that the quadratic fluctuations decouple. We also note that a similar decoupling mechanism for vector and scalar fluctuations was found to be at work for the flow of [20] in [38]. There, it was observed that decoupling was achieved since the gauge field and a (charged) scalar appeared in a gauge invariant combination.

The field redefinition (2.12) removes the scalar fluctuations of δM_{ij} since it removes terms quadratic in first derivatives of δM_{ij} from the lagrangian. The remaining terms are at most linear in first derivatives and of the form $B_{ij}\delta M_{ij}\delta M_{ij} + B^{\widehat{\mu}}_{ij}\delta M_{ij}\partial_{\widehat{\mu}}\delta M_{ij}$ for some space-depended B_{ij} and $B^{\widehat{\mu}}_{ij}$ which are symmetric in i, j. Clearly the derivative-term can be removed by adding an appropriate total derivative so that we are left with a non-dynamical field δM_{ij} corresponding to no physical degrees of freedom. What we have is nothing but a manifestation of the Higgs effect in a curved background. As in flat space-time, the Goldstone bosons corresponding to the broken gauge symmetries are eaten by the gauge bosons which then become massive.

Since we are only interested in the gauge field fluctuations we ignore the scalar fluctuations for the rest of the paper and concentrate on those for the gauge fields which, after the redefinition (2.12), are described by the first line in (2.10)

$$\frac{1}{\sqrt{g}}\mathcal{L}(A)_{\text{quad.}} = -\frac{1}{8}e^{-2(\beta_i+\beta_j)}F^{ij}_{\widehat{\mu}\widehat{\nu}}F^{\widehat{\mu}\widehat{\nu}}_{ij} - \frac{g^2}{4}\sinh^2(\beta_i-\beta_j)A^{ij}_{\widehat{\mu}}A^{\widehat{\mu}}_{ij}\,. \tag{2.13}$$

The second term corresponds to mass terms for the gauge fields, if the scalar fields β_i are not equal. This implies that for general states on the Coulomb branch the bulk gauge symmetry SO(6) is spontaneously broken and, hence, that the $\mathcal{R}$-symmetry group of the field theory on the boundary is reduced accordingly. Notice also that the kinetic term for the gauge fields is not canonically normalized as it gets “dressed” by the scalar fields. This will have important consequences, as we will see.

The equation of motion following from this quadratic action (2.13) is:

$$\delta A^{ij}_{\widehat{\mu}}: \qquad D_{\widehat{\mu}}\left(e^{-2(\beta_i+\beta_j)}F^{\widehat{\mu}\widehat{\nu}}_{ij}\right) - g^2\sinh^2(\beta_i-\beta_j)A^{\widehat{\nu}}_{ij} = 0\,. \tag{2.14}$$

In solving these equations we have to distinguish two cases: first, for the unbroken symmetry (currents), for which $\beta_i = \beta_j$, we can use the gauge symmetry to choose the gauge $A^{ij}_z = 0$. This still allows for restricted gauge transformations with parameters that depend only on the x^μ's, but not on z. Then, the $\widehat{\mu} = z$ component of eqs. (2.14) yields the constraint $\partial_z \partial_\mu A^\mu = 0$ which allows to eliminate unphysical longitudinal modes via a restricted gauge transformation. The equation of motion for the remaining physical (transverse) modes $A^\perp_\mu$ which obey $\partial^\mu A^\perp_\mu = 0$ is the same for all components and can be written as an equation for a scalar field, which we denote by Φ:

$$\partial_z (e^B \partial_z \Phi) + m^2 e^B \Phi = 0\,, \tag{2.15}$$

with the definition

$$B = A - 2(\beta_i + \beta_j)\,. \tag{2.16}$$

To arrive at this equation we have performed a Fourier transform in the x^μ-directions with $k_\mu k^\mu = -m^2$.[3]

For the broken symmetry currents for which $\beta_i \neq \beta_j$ we cannot use a gauge symmetry to eliminate degrees of freedom. In order to calculate the two-point functions we couple the gauge field to an external source by adding $-\frac{1}{2} A^{ij}_{\widehat{\mu}} J^{\widehat{\mu}}_{ij}$ to the gauge field action (2.13). The source is required to be covariantly conserved, i.e. $D^{\widehat{\mu}} J^{ij}_{\widehat{\mu}} = 0$. We choose to decompose the gauge field into transverse modes $A^\perp_\mu$, longitudinal modes $\partial_\mu \xi = A_\mu - A^\perp_\mu$, and the component A_z. The equations of motion (2.14) give

$$\partial_z \left(e^B (\partial_z A^\perp_\mu - \partial_\mu A_z + \partial_z \partial_\mu \xi) \right) + e^B \Box A^\perp_\mu - e^C (\Box A^\perp_\mu + \partial_\mu \xi) = e^{3A} J_z \tag{2.17}$$

and

$$e^B (\Box A_z - \partial_z \Box \xi) - e^C A_z = e^{3A} J_z\,, \tag{2.18}$$

where $\Box = \eta^{\mu\nu} \partial_\mu \partial_\nu$. The above coupled system of equations can be further simplified. By taking the derivatives ∂_μ and ∂_z in (2.17) and (2.18), respectively, adding up the resulting expressions and then using the condition $D^{\widehat{\mu}} J_{\widehat{\mu}} = 0$, we obtain a relation that determines ξ in terms of the component A_z, namely

$$e^C \Box \xi + \partial_z \left(e^C A_z \right) = 0\,, \tag{2.19}$$

where

$$e^C = g^2 e^{3A} \sinh^2(\beta_i - \beta_j) = \frac{1}{4} g^2 (\beta_i - \beta_j)^2 e^{-B}\,. \tag{2.20}$$

The first equality defines C, whereas the second one follows with the help of (2.7) and relates C to B which was defined in (2.16). Using (2.19) to solve for $\Box\xi$ and then substituting back the result into (2.18) we find the equation for the mode A_z, which decouples from the transverse modes:

$$e^B \Box A_z + e^B \partial_z \left(e^{-C} \partial_z \left(e^C A_z \right) \right) - e^C A_z = e^{3A} J_z\,. \tag{2.21}$$

[3]For notational simplicity we did not include indices i, j in defining B in (2.16). Nevertheless it should be kept in mind that different choices for the scalar fields β_i and β_j lead to different values for B.

JHEP12(2000)014

With further manipulations using (2.21), we may cast (2.17) into an equation for the transverse modes

$$e^B\Box A_\mu^\perp + \partial_z\left(e^B\partial_z A_\mu^\perp\right) - e^C A_\mu^\perp = e^{3A}J_\mu^\perp\,, \tag{2.22}$$

where we have defined the transverse current-source as $J_\mu^\perp = (\delta_{\mu\nu} - \partial_\mu\partial_\nu/\Box)J_\nu$. In order to compute the two point functions in momentum space we need solutions of the homogeneous equations (2.21) and (2.22). Actually, after a Fourier transform in the x^μ brane-directions, we can write both equations as an equation for a scalar field

$$\partial_z(e^B\partial_z\Phi) + \left(m^2e^B - \frac{1}{4}g^2(\beta_i-\beta_j)^2e^{-B}\right)\Phi = 0\,, \tag{2.23}$$

where we have dropped the source term. Its effect will be implemented by imposing appropriate boundary conditions to the solutions. For the case of (2.22) the scalar Φ denotes any component of $A_\mu^\perp$. In order to cast (2.21) into the form (2.23), we have used (2.20) and defined $\Phi = e^C A_z$. For $\beta_i = \beta_j$ we recover from (2.23) eq. (2.15) that describes the cases with unbroken symmetry. Hence, for full generality, we may use (2.23) in order to calculate current-current correlators. We will follow the standard procedure of [2, 3] and we will work in euclidean signature unless stated otherwise.

In order to proceed we need a complete set of eigenfunctions of (2.23), which for the examples we will discuss in the next section can be found explicitly and is given in terms of hypergeometric functions. Furthermore, we keep the solutions that blow up at the AdS boundary since they correspond to current operator insertions [2, 3]. Finally, we have to evaluate the on shell-value of the action $\frac{1}{\kappa^2}\int d^5x\mathcal{L}$ with $\frac{1}{\kappa^2} = \frac{N^2}{16\pi^2}$ for solutions Φ of (2.23).[4] We find the boundary term

$$-\lim_{\epsilon\to 0}\frac{N^2}{32\pi^2}e^B\Phi\partial_z\Phi\Big|_{z=\epsilon}^{z_{\max}} \equiv \frac{N^2}{16\pi^2}k^2H(k)\,. \tag{2.24}$$

In order to keep formulas short in later sections we have written out the overall factor $1/\kappa^2$ in the definition of $H(k)$. In order to obtain the correct result we have to normalize $\Phi|_{z=\epsilon} = 1$ and take the limit in (2.24). Re-introducing Lorentz and group theory indices properly, we can present the current-current correlators in momentum space schematically as

$$\langle J_\mu^a(k)J_\nu^b(-k)\rangle = \frac{N^2}{8\pi^2}\delta^{ab}\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)k^4\tilde{G}(k)\,, \tag{2.25}$$

where a group theory factor and the momentum space version of the projector,

[4] The overall normalization is found by carefully keeping track of all the prefactors in the dimensional reduction in the S^5-directions of the ten-dimensional type-IIB action to five dimensions. In particular, $\frac{1}{\kappa^2} = \frac{V_{S^5}}{4\kappa_{10}^2}R^8$. Then using $2\kappa_{10}^2 = (2\pi)^7\alpha'^4g_s^2$, $R^4 = 4\pi g_s\alpha'^2N$ and $V_{S^5} = \pi^3$ we find the result mentioned above.

which guarantees that the amplitude is transverse, have been included. The factor $H(k) \equiv k^2\tilde{G}(k)$ depends also on the adjoint indices a, b, but for reasons similar to those explained in footnote 2 we have not explicitly displayed them.

In the explicit calculations performed later in section 3 we will not use $H(k)$ directly, as defined in (2.24), because the correlator in x-space is too singular to be Fourier transformed to momentum space. However, by using differential regularization one can make sense of such expressions by writing singular functions as derivatives of less singular ones and then defining the Fourier transform by formal partial integrations [41]. In our case we have to take the correlator to be of the form $\sim \Box\Box G(x)$ which is just $k^4\tilde{G}(k)$ in momentum space. Hence, the correlator in x-space becomes

$$\langle J^a_\mu(x) J^b_\nu(0)\rangle = \frac{N^2}{32\pi^4}\delta^{ab}(\Box\delta_{\mu\nu} - \partial_\mu\partial_\nu)\Box G(x)\,, \tag{2.26}$$

where

$$G(x) = \frac{1}{4\pi^2}\int d^4k e^{ik\cdot x}\frac{H(k)}{k^2} = \frac{1}{r}\int_0^\infty dk H(k) J_1(kr)\,, \tag{2.27}$$

with $J_1(kr)$ being a Bessel function.

2.1 Supersymmetric quantum mechanics

In this subsection we want to study general aspects of the fluctuation equation (2.23), before we proceed in section 3 to describe two special cases where calculations can be performed exactly. Writing $\Phi = e^{-B/2}\Psi$ the field equation (2.23) turns into the one-dimensional Schrödinger equation

$$-\Psi'' + V\Psi = m^2\Psi\,, \tag{2.28}$$

with potential

$$V = \frac{1}{4}(B')^2 + \frac{1}{2}B'' + g^2 e^{2(A+\beta_i+\beta_j)}\sinh^2(\beta_i - \beta_j)\,. \tag{2.29}$$

This potential, though not at all obvious, can be cast into a form that appears in supersymmetric quantum mechanics. First, we rewrite it differently using the properties of our solution (2.6) and (2.7) and in particular (2.8) which proves useful in turning derivatives with respect to the variable z into functions of the auxiliary function F only:

$$V = \frac{g^4 f^{1/2}}{64}\left[8\sum_{i=1}^{6}\frac{1}{(F-b_i)^2} - \left(\sum_{i=1}^{6}\frac{1}{F-b_i}\right)^2\right]. \tag{2.30}$$

Comparing with [13, eq. (4.16)] (after setting in there the parameter $\Delta = 4$) we find that this can be written solely in terms of the conformal factor in the metric ansatz (2.5)

$$V = \frac{9}{4}A'^2 - \frac{3}{2}A''\,. \tag{2.31}$$

This potential[5] has the same form as the potential appearing in supersymmetric quantum mechanics [42, 43] with superpotential $W = -3/2A'$. In fact, it is the supersymmetric partner of the potential

$$V_s = \frac{9}{4}A'^2 + \frac{3}{2}A'' , \tag{2.32}$$

that appeared in studies of 2-point functions for scalar fields or transverse graviton fluctuations [9, 11, 12, 13, 35, 44]; the relation of (2.32) to supersymmetric quantum mechanics in the context of gauged supergravity was first hinted in [11] and explicitly noted in [44]. Note that, the Schrödinger problem is universal and does not depend on the indices i, j of the gauge currents. Consequently, the mass spectrum is the same irrespectively of whether it is associated to currents corresponding to broken or unbroken symmetries. Instead, the wavefunction Φ does depend on the indices i, j through the explicit dependence on them of the conformal factor B defined in (2.16) (cf. footnote 3).

It is well known from the general theory of supersymmetric quantum mechanics that the spectra of superpartner potentials, such as (2.31) and (2.32), are identical except for a zero mode. However, in our case such a mode is not normalizable due to the asymptotic behavior of the function $A(z)$ as $z \to 0$ and, therefore, is not included in the spectrum. Hence, the spectra of current fluctuations, corresponding to (2.31) and those for dilaton and transverse graviton fluctuations, corresponding to (2.32), exactly coincide, as advertised in the introduction. We note, that related observations concerning a SO(3) invariant sector of 5d gauged supergravity and a particular Coulomb branch flow have been made in [38].[6]

The analysis of the qualitative features of the spectrum can be done in a similar fashion as in the case of the superpartner arising in the case of scalar correlators [11, 12]. At the boundary $z = 0$ the potential goes to $+\infty$ as $V \simeq \frac{3}{4z^2}$. The behavior in the interior depends on the number n of constants of integration b_i that equal the maximum constant among them, b_1. We follow closely the discussion of [11, 13] to which we refer for further details. For $n = 4, 5$ the range of z necessarily extends to $+\infty$, i.e. $0 \le z < \infty$, corresponding to $F = b_1$. We find that, for $n = 5$, the potential goes to zero as $z \to \infty$ and the spectrum is continuous. For $n = 4$ the potential approaches a constant value, as $z \to \infty$, which is given by $V_{\min} = \frac{g^4}{4} f_0^{1/2}$. Therefore, although the spectrum is continuous, there is a mass gap whose squared value is given by the minimum of the potential. For $n = 5$ the potential behaves as

$$n = 5 : \qquad V_5 \simeq \frac{15/4}{z^2} , \qquad \text{as} \quad z \longrightarrow \infty . \tag{2.33}$$

[5]An alternative way to prove the equivalence of the potentials (2.29) and (2.31) is to use the differential equation obeyed by the β_i's, namely $\beta_i' = A' + \frac{g}{2}e^{A+2\beta_i}$ [11].

[6]The authors of [38] informed us that their arguments concerning graviphotons are actually broader and include all massive cases where $U(1)_R$ is broken.

JHEP12(2000)014

For $n = 1, 2, 3$ the potential goes to $+\infty$ as $F \to b_1$ and therefore the spectrum must be discrete. Therefore there should be a maximum value for z, denoted by $z_{\max}$, that is determined by solving the algebraic equation $F(z_{\max} g^2) = b_1$. We find the behaviour

$$
\begin{aligned}
n = 1, 2, 3: \qquad V_n &\simeq \frac{C_n}{(z - z_{\max})^2}, \qquad \text{as} \quad z \longrightarrow z_{\max}^-, \\
C_n &= \frac{4}{(4-n)^2} - \frac{1}{4}.
\end{aligned} \tag{2.34}
$$

For more details on the full structure of the potentials (2.31) and (2.32), which generically can be written using elliptic functions, the reader is referred to the original literature [11, 13]. In the two special cases, to which we turn now in section 3, all computations and results can be written in terms of elementary functions.

3. The 2-point function

In the previous section we introduced all necessary ingredients for the calculation of correlators of symmetry currents and pointed out the relation between supersymmetric quantum mechanics and the fluctuation equations. In this section we want to use these results and apply them to two specific backgrounds worked out in [9, 10, 11]. These backgrounds correspond to distributions of D3-branes on a disc or a three-sphere [5, 6] and they both break the bulk gauge symmetry down to $\mathrm{SO}(2) \times \mathrm{SO}(4)$. The broken symmetries form the coset $\frac{\mathrm{SO}(6)}{\mathrm{SO}(2)\times\mathrm{SO}(4)}$. On the dual field theory side these backgrounds correspond to states on the Coulomb branch of $\mathcal{N} = 4$ SYM theory with reduced $\mathcal{R}$-symmetry. In the following we will calculate the correlators in momentum and position spaces.

3.1 Distribution of D3-branes on a three-sphere

We begin our exactly solvable examples with the case of a model representing D3-branes uniformly distributed on a three-sphere. The expressions for the metric and the scalar fields have been given in [9, 11]. The five-dimensional metric (2.5) has the conformal factor

$$
e^{2A} = \frac{r_0^2}{R^2} \frac{\cos^{2/3} u}{\sin^2 u}, \qquad 0 \le u \le \frac{\pi}{2}, \tag{3.1}
$$

where we have defined for notational purposes the dimensionless variable $u = r_0 z / R^2$. The parameter r_0 actually plays the rôle of the radius of the three-sphere. The AdS_5 boundary corresponds to $u = 0$, whereas at $u = \pi/2$ there is a naked curvature singularity. This is however naturally interpreted, from a string theoretical point of view, as the location of the distribution of the D3-branes on the three-sphere.

The profiles of the scalar fields are

$$
e^{2\beta_1} = e^{2\beta_2} = \cos^{-4/3} u, \qquad e^{2\beta_3} = \cdots = e^{2\beta_6} = \cos^{2/3} u. \tag{3.2}
$$

From a ten-dimensional view point, these scalars deform the five-sphere line element that appears in the D3-brane solution in such a way that the subgroup $SO(2)\times SO(4)$ of the isometry group SO(6) is preserved. The Schrödinger potential (2.31) is found to be

$$V=\frac{r_0^2}{R^4}\left(-1+\frac{3}{\sin^2 2u}\right). \tag{3.3}$$

It is not difficult to show that a complete orthonormal set of solutions to the corresponding Schrödinger equation is given by

$$\Psi_n=\sqrt{\frac{2n(n+1)}{n+1}\frac{r_0}{R^2}}\,\frac{\cos^{3/2}u}{\sin^{1/2}u}\,P_n^{(-1,1)}(\cos 2u)\,,\qquad 0\le u\le\frac{\pi}{2}\,,\qquad n=1,2,\ldots\,, \tag{3.4}$$

where the $P_n^{(-1,1)}$'s are Jacobi polynomials, provided that the spectrum is given by

$$m_n^2=\frac{4r_0^2}{R^4}n(n+1)\,,\qquad n=1,2,\ldots\,. \tag{3.5}$$

Note that the case with $n=0$, giving rise to a zero-mass eigenvalue, is not included in the spectrum since the corresponding Schrödinger norm diverges. The eigenvalues (3.5) coincide with those found for dilaton fluctuations in [9, 10] using the same background as here, in agreement with our general discussion in section 2. Also the n-dependent overall constant in (3.4) has been chosen such that the Ψ_n's are normalized to one.

The conformal factor appearing in the equation of the fluctuations (2.23) is:

$$e^B=\frac{r_0}{R}\times\begin{cases}\dfrac{\cos^3 u}{\sin u}\,, & i,j=1,2\,,\\[2mm] \dfrac{1}{\sin u\cos u}\,, & i,j=3,4,5,6\,,\\[2mm] \dfrac{\cos u}{\sin u}\,, & i=1,2\,,\quad j=3,4,5,6\,.\end{cases} \tag{3.6}$$

3.1.1 The 2-point functions

Using (2.23), (3.2) and (3.6) we find the wave equation for the transverse modes of the gauge field in the unbroken SO(2) subgroup, the coset and the unbroken SO(4):

$$\begin{aligned}(1-x)(x^2\Phi')'-\frac{\tilde{k}^2}{4}\Phi&=0\,,\\ (1-x)(x\Phi')'-\frac{\tilde{k}^2}{4}\Phi-\frac{1-x}{4x}\Phi&=0\,,\qquad x\equiv\cos^2 u\in[0,1]\,,\\ x(1-x)\Phi''-\frac{\tilde{k}^2}{4}\Phi&=0\,,\end{aligned} \tag{3.7}$$

where the prime denotes derivatives with respect to x and $\tilde{k}^2=R^4/r_0^2k_\mu k^\mu$, i.e. is the length-square of the four-vector k^μ rescaled for notational convenience with the indicated factor.

The wave-functions that blow up at the boundary at $x = 1$ and are regular at the singularity at $x = 0$ are given in terms of a hypergeometric function as[7]

$$\Phi = \Gamma\left(\frac{3+\Delta}{2}\right)\Gamma\left(\frac{3-\Delta}{2}\right)x^{\lambda}F\left(\frac{1+\Delta}{2},\frac{1-\Delta}{2},2,x\right), \tag{3.8}$$

where $\Delta = \sqrt{1-\tilde{k}^2}$, and where we have introduced the parameter $\lambda = 0, 1/2$ and 1 for the currents corresponding to the unbroken SO(2), the broken coset and the unbroken SO(4) symmetries, respectively. The proportionality constant in (3.8) has been fixed such that $\Phi(1) = 1$ and hence at the boundary the solution becomes proportional to a δ-function, i.e. fully localized operator insertion. It is interesting to note that the wavefunctions Φ in all three cases differ only by different powers of x. This is related, as we have seen, to the fact that the mass spectra for broken and unbroken currents are identical. From (3.8) we extract

$$\begin{aligned} H(\tilde{k}) &= \frac{1-\lambda}{\tilde{k}^2} + \frac{1}{4}\left(\psi\left(\frac{1+\Delta}{2}\right)+\psi\left(\frac{1-\Delta}{2}\right)+2\gamma\right) \\ &= -\frac{\lambda}{\tilde{k}^2} + \frac{1}{2}\sum_{n=1}^{\infty}\frac{2n+\tilde{k}^2}{n(4n(n+1)+\tilde{k}^2)}, \end{aligned} \tag{3.9}$$

which has a discrete spectrum of poles at $\tilde{k}^2 = -4n(n+1)$, $n = 1, 2, \ldots$, corresponding precisely to the mass eigenvalues (3.5). However, if $\lambda \neq 0$, there is an additional pole at $\tilde{k}^2 = 0$. We will comment on this in various places below.

The three correlators differ only in the coefficient of the $1/\tilde{k}^2$ term. In the case of scalar correlators this would just give a contact term and could be ignored, but in the case of the symmetry-current correlators this has important consequences as we will explain shortly. Using (2.27) we obtain the following exact expression for the function $G(x)$ in the correlator (2.26):

$$G(x) = \lambda\frac{r_0^2}{2R^4}\ln r + \frac{r_0}{2R^2 r}\sum_{n=1}^{\infty}\frac{2n+1}{\sqrt{n(n+1)}}K_1\left(2\sqrt{n(n+1)}\frac{rr_0}{R^2}\right), \tag{3.10}$$

where K_1 denotes the modified Bessel function and in writing the term containing $\ln r$ we discarded an infinite constant. We have also dropped a $1/r^2$ term, which, since $\Box 1/r^2 \sim \delta^{(4)}(r)$, contributes only contact terms to the correlator which we consistently ignore. Hence, we find

$$\Box G(x) = \lambda\frac{r_0^2}{R^4 r^2} + \frac{2r_0^3}{R^6 r}\sum_{n=1}^{\infty}(2n+1)\sqrt{n(n+1)}K_1\left(2\sqrt{n(n+1)}\frac{rr_0}{R^2}\right). \tag{3.11}$$

[7] Throughout the paper we will make use of special functions and their properties following the conventions of [45].

Let us perform the consistency check that for small r, or equivalently, in the limit $r_0 \to 0$, we should recover the conformal result. The dominant contribution in this limit comes from the infinite sum which can be approximated by an integral

$$G(x) = \frac{1}{2r^2} \int_1^{1/r} \frac{dn}{n} + \cdots \simeq -\frac{1}{4r^2} \ln r^2 \,, \qquad \text{as} \quad r \longrightarrow 0 \,. \tag{3.12}$$

This gives rise to

$$\Box G(x) \simeq \frac{1}{r^4} \,, \qquad \text{as} \quad r \longrightarrow 0 \,, \tag{3.13}$$

which in turn, gives a $1/r^6$ fall off for the correlator (2.26) at short distances. As expected, this coincides with the result in the conformal case (see, for instance, [32, eq. (30)]).

The behavior of $G(x)$ for large r is easily found from the asymptotic expansion of the modified Bessel function. For large r each separate term in the infinite sum behaves as $e^{-m_n r}/r^{3/2}$, where m_n are the mass eigenvalues in (3.5) and hence gives rise to an exponential fall off. Keeping the two most dominant contributions in the r.h.s. of (3.10) we obtain

$$G(x) \;\simeq\; \lambda \frac{r_0^2}{2R^4} \ln r + \frac{3\sqrt{\pi}}{8\sqrt{2}} \left(\frac{R^2}{r_0 r} \right)^{3/2} e^{-2\sqrt{2} r_0 r / R^2} \,, \qquad \text{as} \quad r \longrightarrow \infty \,. \tag{3.14}$$

For the cases corresponding to the broken coset currents and the unbroken SO(4) currents we have $\lambda \neq 0$ and therefore the dominant contribution for large r comes from the first term in (3.14). When substituted into the correlator in (2.26) it produces a contact term, which we drop, and a term of the form

$$\langle J_\mu^a(x) J_\nu^b(0) \rangle \simeq \lambda \delta^{ab} \frac{N^2}{4\pi^2} \frac{r_0^2}{R^4} \frac{1}{r^6} \left(r^2 \delta_{\mu\nu} - 4 x_\mu x_\nu \right) , \qquad \text{as} \quad r \longrightarrow \infty \,. \tag{3.15}$$

This term decays only with the forth power of the distance and at first sight it might be tempting to interpret it as arising from the massless Goldstone boson associated with the broken symmetry.[8] From a physical point of view there are several problems with such an interpretation: first, this term does not appear on equal footing for all three types of currents although they reside in the same supersymmetry multiplet. Its existence might seemingly be acceptable or even desirable for the broken symmetry, but this term also appears for the unbroken SO(4)-symmetry currents. We also know from section 2 that the gauge fields dual to the broken currents become massive via the Higgs mechanism and, therefore, are not expected to produce any massless states. Second, the pole of the massless state corresponds to a non-normalizable mode and it is not expected to show up in the two-point function. The most plausible solution seems to be that these poles are actually unphysical and should be dropped from the

[8] Work on the AdS/CFT correspondence and the Goldstone bosons has been reported using a different model in [21].

correlators. Note that a similar problem was found in [36] for the two-point function of active scalars in the same backgrounds we are discussing here. The mysterious massless poles in that paper were later shown to be absent if a different prescription for the correlators is used [37]. It seems likely, although we have not checked, that an improved prescription would resolve the puzzle in our case as well.

3.2 Distribution of D3-branes on a disc

Our second exactly solvable model represents D3-branes uniformly distributed on a disc of radius r_0. The expressions for the metric and the scalar fields have been given in [9, 11]. The five-dimensional metric (2.5) has the conformal factor

$$e^{2A} = \frac{r_0^2}{R^2}\frac{\cosh^{2/3} u}{\sinh^2 u}\,, \qquad 0 \le u < \infty\,, \tag{3.16}$$

where as before $u = r_0 z/R^2$. The scalar fields are given by

$$e^{2\beta_1} = \cdots = e^{2\beta_4} = \cosh^{2/3} u\,, \qquad e^{2\beta_5} = e^{2\beta_6} = \cosh^{-4/3} u\,. \tag{3.17}$$

As before, from a ten-dimensional type-IIB view point, these scalars deform the five-sphere line element that appears in the D3-brane solution in such a way that the subgroup $SO(2) \times SO(4)$ of the isometry group SO(6) is preserved. The Schrödinger potential (2.31) becomes

$$V = \frac{r_0^2}{R^4}\left(1 + \frac{3}{\sinh^2 2u}\right). \tag{3.18}$$

The energy spectrum for this potential is continuous and has a mass gap

$$m^2 \ge \frac{r_0^2}{R^4}\,. \tag{3.19}$$

As before the zero mode corresponds to a non-normalizable wavefunction.

The conformal factor appearing in the equation of the fluctuations (2.23) is:

$$e^B = \frac{r_0}{R} \times \begin{cases} \dfrac{\cosh^3 u}{\sinh u}\,, & i,j = 1,2\,, \\ \dfrac{1}{\sinh u \cosh u}\,, & i,j = 3,4,5,6\,, \\ \dfrac{\cosh u}{\sinh u}\,, & i = 1,2\,, \quad j = 3,4,5,6\,. \end{cases} \tag{3.20}$$

3.2.1 The 2-point function

The wave equation (2.23) for the gauge fields of the unbroken SO(2), the broken coset and the unbroken SO(4) symmetries, respectively, are:

$$\begin{aligned} x^2(1-x)\Phi'' - \frac{\tilde{k}^2}{4}\Phi &= 0\,, \\ x(1-x)(x\Phi')' - \frac{k^2}{4}\Phi - \frac{1}{4}(1-x)\Phi &= 0\,, \qquad x \equiv \frac{1}{\cosh^2 u} \in [0,1]\,, \\ (1-x)(x^2\Phi')' - \frac{\tilde{k}^2}{4}\Phi &= 0\,, \end{aligned} \tag{3.21}$$

where, as before, $\tilde{k}^2 = k^2R^4/r_0^2$. The properly normalized solution that is also regular in the interior is

$$\Phi = \frac{\Gamma((1+\Delta)/2)\Gamma((3+\Delta)/2)}{\Gamma(1+\Delta)} x^{(1+\Delta)/2-\lambda} F\left(\frac{\Delta-1}{2}, \frac{\Delta+1}{2}, 1+\Delta, x\right) , \qquad (3.22)$$

where $\Delta = \sqrt{\tilde{k}^2+1}$ and similarly to before, the parameter $\lambda = 0, 1/2$ and 1 for the currents corresponding to SO(2), to the coset and to SO(4), respectively. From this we obtain

$$\begin{aligned} H(\tilde{k}) &= \frac{\lambda-1}{k^2} + \frac{1}{2}\left(\psi\left(\frac{1+\Delta}{2}\right)+\gamma\right) \\ &= \frac{\lambda-1}{k^2} + \frac{1}{2}\int_0^\infty dt \frac{e^{-t}-e^{-\frac{\Delta+1}{2}t}}{1-e^{-t}} \end{aligned} \qquad (3.23)$$

and then

$$G(x) = \frac{1}{2}(1-\lambda)\frac{r_0^2}{R^4}\ln r + \frac{1}{2r^2}\int_0^\infty dy \frac{y}{\sinh y}\frac{e^{-\sqrt{y^2+r_0^2r^2/R^4}}}{\sqrt{y^2+r_0^2r^2/R^4}} . \qquad (3.24)$$

Using this result it can be easily seen that the short distance behavior of the propagator is the same as in the conformal case and in particular (3.12) is recovered. At large distances one finds that the two most dominant terms are

$$G(x) \simeq \frac{1}{2}(1-\lambda)\frac{r_0^2}{R^4}\ln r + \frac{\pi^2}{8}\frac{R^2}{r_0r^3}e^{-r_0r/R^2} , \qquad \text{as} \quad r \longrightarrow \infty , \qquad (3.25)$$

where naturally the range of the Yukawa-term is set by the mass gap in (3.19). Hence, for the case where $\lambda \neq 1$, corresponding to the cases of the broken coset and the unbroken SO(2) symmetries, the first term dominates for large r giving a contribution to the correlator similar to (3.15), but with λ replaced by $1-\lambda$. For similar reasons to those that we outlined for the case of the sphere-distribution of D3-branes after (3.15), the interpretation of such a term as being related to the Goldstone bosons is problematic and we believe that they are unphysical.

4. Discussion

In this letter we studied $\mathcal{R}$-symmetry current correlators in certain states on the Coulomb branch of $\mathcal{N}=4$ SYM using the standard description of the AdS/CFT correspondence. The surprising result is that the spectra derived from the analytic structure of the correlators agree with spectra of other operators corresponding to dilaton and to the transverse graviton fluctuations. Furthermore, it turned out that the spectra are identical and do not depend on whether they are in the unbroken part of the left over global symmetry or reside in the coset, except for certain zero-mass poles which do depend on the sector. These poles give rise to a $1/r^4$ fall off of the correlators at large distances, the behavior expected of massless scalars, but we did not find good physical reasons to identify them with Goldstone bosons of the broken symmetry currents. We rather think that these poles are unphysical since they correspond to non-normalizable states and are inconsistent with the fact that the currents are all in the same supersymmetry multiplet.

Rephrasing the fluctuation equations into a supersymmetric quantum mechanics problem we found that they all fall into the same universality class and, furthermore, the Schrödinger potential are the supersymmetric partner potentials arising from the dilaton or from the transverse graviton fluctuations, which are identical. This indicates that all fluctuations in such backgrounds fall into the same class of supersymmetric quantum mechanics problems.

To obtain a more complete picture including the Goldstone bosons one probably has to include additional modes that live on the D3-branes which create the singularity in the infrared. In our set up with a continuous distribution of branes this seems a formidable task, and as a starting point it seems more feasible to study simpler examples, e.g. two stacks of coinciding branes or a single test brane separated from a stack of branes, in which case one would readily know the additional modes and their respective couplings to the bulk fields. We leave these issues for future work.

It will also be interesting to investigate current-correlators using solutions of $D = 7$ and $D = 4$ gauged supergravity that are dual to the (2,0) theories in six dimensions and the three-dimensional theories with sixteen supercharges, respectively, on the Coulomb branch. For a class of such backgrounds corresponding to a scalar-gravity sector analogous the one used in the present paper the most general solution has been found and is very similar to that in (2.5)–(2.8) [13] (see also [12]). The spectrum, of fluctuations corresponding to a massless scalar has been also exhaustively studied and in some cases the computations can be performed explicitly [13]. Similarly to the present paper, in these cases as well, it is quite plausible that the current-correlators and the associated spectra are related via supersymmetric quantum mechanics to those of the massless scalar.

Acknowledgments

A.B. would like to thank Y. Oz, J. Sonnenschein and S. Yankielowicz for discussions, and the universities of Tel-Aviv, Neuchâtel and Ludwig-Maximilians in Munich for hospitality and financial support. K.S. would like to thank the Theory Division at CERN for hospitality and financial support during a considerable part of this research. The research of K.S. was supported by the European Union under contracts TMR-ERBFMRX-CT96-0045 and -0090, by the Swiss Office for Education and Science, by the Swiss National Foundation and by the contract HPRN-CT-2000-00122.

References

[1] J. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200].

[2] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109].

[3] E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150]; *Anti-de Sitter space, thermal phase transition and confinement in gauge theories*, *Adv. Theor. Math. Phys.* **2** (1998) 505 [hep-th/9803131].

[4] J.A. Minahan and N.P. Warner, *Quark potentials in the Higgs phase of large-N supersymmetric Yang-Mills theories*, *J. High Energy Phys.* **06** (1998) 005 [hep-th/9805104].

[5] P. Kraus, F. Larsen and S.P. Trivedi, *The Coulomb branch of gauge theory from rotating branes*, *J. High Energy Phys.* **03** (1999) 003 [hep-th/9811120].

[6] K. Sfetsos, *Branes for Higgs phases and exact conformal field theories*, *J. High Energy Phys.* **01** (1999) 015 [hep-th/9811167].

[7] I.R. Klebanov and E. Witten, *AdS/CFT correspondence and symmetry breaking*, *Nucl. Phys.* **B 556** (1999) 89 [hep-th/9905104].

[8] A. Kehagias and K. Sfetsos, *On running couplings in gauge theories from type-IIB supergravity*, *Phys. Lett.* **B 454** (1999) 270 [hep-th/9902125]; *On asymptotic freedom and confinement from type-iib supergravity*, *Phys. Lett.* **B 456** (1999) 22 [hep-th/9903109];
S.S. Gubser, *Dilaton-driven confinement*, hep-th/9902155;
H. Liu and A.A. Tseytlin, *D3-brane D-instanton configuration and $N = 4$ super YM theory in constant self-dual background*, *Nucl. Phys.* **B 553** (1999) 231 [hep-th/9903091];
N.R. Constable and R.C. Myers, *Exotic scalar states in the AdS/CFT correspondence*, *J. High Energy Phys.* **11** (1999) 020 [hep-th/9905081].

[9] D.Z. Freedman, S.S. Gubser, K. Pilch and N.P. Warner, *Continuous distributions of d3-branes and gauged supergravity*, *J. High Energy Phys.* **07** (2000) 038 [hep-th/9906194].

[10] A. Brandhuber and K. Sfetsos, *Wilson loops from multicentre and rotating branes, mass gaps and phase structure in gauge theories*, hep-th/9906201.

[11] I. Bakas and K. Sfetsos, *States and curves of five-dimensional gauged supergravity*, *Nucl. Phys.* **B 573** (2000) 768 [hep-th/9909041].

[12] M. Cvetič, S.S. Gubser, H. Lu and C.N. Pope, *Symmetric potentials of gauged supergravities in diverse dimensions and coulomb branch of gauge theories*, *Phys. Rev.* **D 62** (2000) 086003 [hep-th/9909121].

JHEP12(2000)014

[13] I. Bakas, A. Brandhuber and K. Sfetsos, *Domain walls of gauged supergravity, M-branes and algebraic curves*, `hep-th/9912132`.

[14] L. Girardello, M. Petrini, M. Porrati and A. Zaffaroni, *Novel local CFT and exact results on perturbations of $N = 4$ super Yang-Mills from AdS dynamics*, *J. High Energy Phys.* **12** (1998) 022 [`hep-th/9810126`].

[15] J. Distler and F. Zamora, *Non-supersymmetric conformal field theories from stable anti-de Sitter spaces*, *Adv. Theor. Math. Phys.* **2** (1999) 1405 [`hep-th/9810206`].

[16] A. Khavaev, K. Pilch and N.P. Warner, *New vacua of gauged $N = 8$ supergravity in five dimensions*, *Phys. Lett.* **B 487** (2000) 14 [`hep-th/9812035`].

[17] A. Karch, D. Lust and A. Miemiec, *New $N = 1$ superconformal field theories and their supergravity description*, *Phys. Lett.* **B 454** (1999) 265 [`hep-th/9901041`].

[18] D.Z. Freedman, S.S. Gubser, K. Pilch and N.P. Warner, *Renormalization group flows from holography supersymmetry and a c-theorem*, `hep-th/9904017`.

[19] K. Behrndt, *Domain walls of $D = 5$ supergravity and fixpoints of $N = 1$ super Yang-Mills*, *Nucl. Phys.* **B 573** (2000) 127 [`hep-th/9907070`].

[20] L. Girardello, M. Petrini, M. Porrati and A. Zaffaroni, *The supergravity dual of $N = 1$ super Yang-Mills theory*, *Nucl. Phys.* **B 569** (2000) 451 [`hep-th/9909047`].

[21] J. Distler and F. Zamora, *Chiral symmetry breaking in the AdS/CFT correspondence*, *J. High Energy Phys.* **05** (2000) 005 [`hep-th/9911040`].

[22] J. Polchinski and M.J. Strassler, *The string dual of a confining four-dimensional gauge theory*, `hep-th/0003136`.

[23] K. Pilch and N.P. Warner, *$N = 2$ supersymmetric rg flows and the IIB dilaton*, *Nucl. Phys.* **B 594** (2001) 209 [`hep-th/0004063`].

[24] A. Brandhuber and K. Sfetsos, *An $N = 2$ gauge theory and its supergravity dual*, *Phys. Lett.* **B 488** (2000) 373 [`hep-th/0004148`].

[25] N. Evans and M. Petrini, *AdS RG-flow and the super Yang-Mills cascade*, *Nucl. Phys.* **B 592** (2000) 129 [`hep-th/0006048`].

[26] K. Pilch and N.P. Warner, *$N = 1$ supersymmetric renormalization group flows from IIB supergravity*, `hep-th/0006066`.

[27] I.R. Klebanov and M.J. Strassler, *Supergravity and a confining gauge theory: duality cascades and χSB-resolution of naked singularities*, *J. High Energy Phys.* **08** (2000) 052 [`hep-th/0007191`].

[28] A. Buchel, A.W. Peet and J. Polchinski, *Gauge dual and noncommutative extension of an $N = 2$ supergravity solution*, `hep-th/0008076`.

JHEP12(2000)014

[29] N. Evans, C.V. Johnson and M. Petrini, *The enhancon and $N = 2$ gauge theory/gravity RG flows*, *J. High Energy Phys.* **10** (2000) 022 [hep-th/0008081].

[30] M. Pernici, K. Pilch and P. van Nieuwenhuizen, *Gauged $N = 8$ $D = 5$ supergravity*, *Nucl. Phys.* **B 259** (1985) 460.

[31] M. Gunaydin, L.J. Romans and N.P. Warner, *Gauged $N = 8$ supergravity in five-dimensions*, *Phys. Lett.* **B 154** (1985) 268;
Compact and noncompact gauged supergravity theories in five-dimensions, *Nucl. Phys.* **B 272** (1986) 598.

[32] D.Z. Freedman, S.D. Mathur, A. Matusis and L. Rastelli, *Correlation functions in the CFT_d/AdS_{d+1} correspondence*, *Nucl. Phys.* **B 546** (1999) 96 [hep-th/9804058].

[33] E. D'Hoker and D.Z. Freedman, *Gauge boson exchange in AdS_{d+1}*, *Nucl. Phys.* **B 544** (1999) 612 [hep-th/9809179].

[34] D. Anselmi, L. Girardello, M. Porrati and A. Zaffaroni, *A note on the holographic beta and c functions*, *Phys. Lett.* **B 481** (2000) 346 [hep-th/0002066].

[35] A. Brandhuber and K. Sfetsos, *Non-standard compactifications with mass gaps and Newton's law*, *J. High Energy Phys.* **10** (1999) 013 [hep-th/9908116].

[36] O. DeWolfe and D.Z. Freedman, *Notes on fluctuations and correlation functions in holographic renormalization group flows*, hep-th/0002226.

[37] G. Arutyunov, S. Frolov and S. Theisen, *A note on gravity-scalar fluctuations in holographic RG flow geometries*, *Phys. Lett.* **B 484** (2000) 295 [hep-th/0003116].

[38] M. Bianchi, O. DeWolfe, D.Z. Freedman and K. Pilch, *Anatomy of two holographic renormalization group flows*, hep-th/0009156.

[39] M.S. Costa, *A test of the AdS/CFT duality on the Coulomb branch*, *Phys. Lett.* **B 482** (2000) 287 [hep-th/0003289]; *Absorption by double-centered d3-branes and the coulomb branch of $N = 4$ sym theory*, *J. High Energy Phys.* **05** (2000) 041 [hep-th/9912073];
R.C. Rashkov and K.S. Viswanathan, *Correlation functions in the Coulomb branch of $N = 4$ SYM from AdS/CFT correspondence*, *Phys. Rev.* **D 62** (2000) 046009 [hep-th/9911160].

[40] M. Cvetič, H. Lu, C.N. Pope, A. Sadrzadeh and T.A. Tran, *Consistent* SO(6) *reduction of type-IIB supergravity on S_5*, *Nucl. Phys.* **B 586** (2000) 275 [hep-th/0003103].

[41] D.Z. Freedman, K. Johnson and J.I. Latorre, *Differential regularization and renormalization: a new method of calculation in quantum field theory*, *Nucl. Phys.* **B 371** (1992) 353.

[42] E. Witten, *Dynamical breaking of supersymmetry*, *Nucl. Phys.* **B 188** (1981) 513.

[43] For a review see, F. Cooper, A. Khare and U. Sukhatme, *Supersymmetry and quantum mechanics*, *Phys. Rep.* **251** (1995) 267 [hep-th/9405029].

[44] O. DeWolfe, D.Z. Freedman, S.S. Gubser and A. Karch, *Modeling the fifth dimension with scalars and gravity*, *Phys. Rev.* **D 62** (2000) 046008 [hep-th/9909134].

[45] I.S. Gradshteyn and I.M. Ryzhik, *Table of integrals, series and products*, fifth edition, Academic Press, New York 1994.